

Half-quantum vortices in strongly correlated Bose liquids.

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Abstract

We discuss the structure of a vortex in a superfluid Bose liquid with a suppressed Bose-Einstein condensate and an intensive pair correlated condensate. The vortex represents the pair of half-quantum vortices topologically confined by the soliton.

In strongly interacting and strongly correlated Bose liquid – superfluid ^4He – the density of the Bose-condensate is small compared to the total mass density of the liquid, $\rho_0 \ll \rho$. This fact leads to speculations that the rest part of the liquid can be described in terms of the Cooper-like pair-correlated condensate (the most recent discussion of this idea is in Ref. [1]; references to the older papers can be found in [2]). We would like to point out that if this idea is correct, it must have a pronounced effect on the core structure of the elementary vortex, which must be non-axisymmetric.

Let us accept that the Cooper-like pair condensate is dominating and gives the dominating contribution to the superfluid density. Then the phenomenological free energy describing interacting pair-

and Bose- condensates, which contains all the relevant physics, can be written as follows

$$F = \frac{1}{2}\rho\mathbf{v}_s^2 + \epsilon(\rho) + F\{\Psi\} . \quad (1)$$

Here \mathbf{v}_s

$$\mathbf{v}_s = \frac{\hbar}{2m}\nabla\phi , \quad (2)$$

is the superfluid velocity expressed in terms of the of pair condensate phase ϕ ; and ρ is the total mass density of the liquid (we consider $T \rightarrow 0$). The single-particle Bose-condensate produces the small correction which as we assume can be described in terms of the Ginzburg-Landau functional, the last term in Eq.(1). The Galilean invariant Ginzburg-Landau functional for the wave function of Bose condensate $\Psi = |\Psi|e^{i\Phi}$ has the form

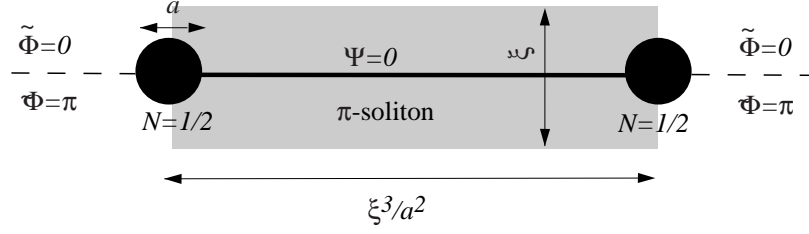
$$F\{\Psi\} = \frac{\beta}{2\rho} \left(|\Psi|^2 - \rho_0\right)^2 + \frac{\hbar^2}{2m^2} |(-i\nabla - m\mathbf{v}_s)\Psi|^2 + \tilde{\alpha}|\Psi|^2 \sin^2\left(\Phi - \frac{\phi}{2}\right). \quad (3)$$

Here $\rho_0 \ll \rho$ is the density of the Bose condensate in equilibrium, which is much smaller than the total density. The last term in Eq.(3) is the Josephson interaction coupling the two condensates; it provides the phase coherence of the two condensates in equilibrium, when $\Phi = \frac{\phi}{2}$ and thus the two condensates have the common superfluid velocity.

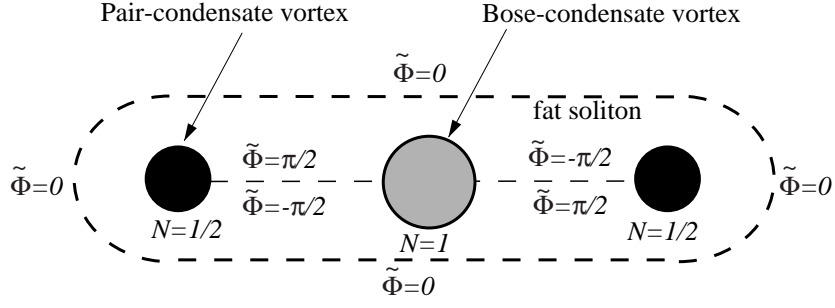
We are interested in the structure of the $N = 1$ vortex in this mixture of two condensates. Around the $N = 1$ vortex the phase of the Bose-condensate Φ changes by 2π , while the phase ϕ of the pair-condensate wave-function changes by 4π . In other words, from the point of view of the pair condensate, the $N = 1$ vortex is doubly quantized. If $\rho_0 = 0$, i.e. in case of pure pair condensate, the elementary vortices of pair condensate have 2π winding of ϕ , and thus they have twice smaller elementary circulation $\kappa_2 = \frac{1}{2}\kappa_0$, where $\kappa_0 = 2\pi\hbar/m$. From the point of view of the Bose condensate they represent vortices with $N = 1/2$ (discussion of the half-quantum vortices – Alice strings – can be found in the book [3]). For nonzero but small $\rho_0 \ll \rho$, the half-quantum vortices are combined into pairs forming $N = 1$ vortices (Fig. 1). Let us consider the structure of such a vortex molecule.

Introducing

$$\tilde{\Phi} = \Phi - \frac{\phi}{2} , \quad \tilde{\Psi} = |\Psi|e^{i\tilde{\Phi}} \quad (4)$$



(a)



(b)

Figure 1: Asymmetric vortex as pair of half-quantum vortices. (a) Vortex structure in case of strong pinning of the Bose-condensate phase Φ by the phase ϕ of the pair condensate. Half-quantum vortices of the pair condensate are confined by π -soliton of the Bose-condensate. Within the soliton the Bose-condensate order parameter crosses zero. (b) Weak-pinning case. The amplitude of the Bose-condensate order parameter has equilibrium value $\sqrt{\rho_0}$ everywhere except for the core of the $N = 1$ vortex in the Bose-condensate. In both Figures thin dashed lines terminating on half-quantum vortices are lines where the phase $\tilde{\Phi}$ has a π -jump required by the Aharonov-Bohm effect experienced by the Bose condensate in the presence of the half-quantum vortices of pair condensate.

one obtains

$$F\{\tilde{\Psi}\} = \frac{\beta}{2\rho} \left(|\tilde{\Psi}|^2 - \rho_0 \right)^2 + \frac{1}{2} \frac{\hbar^2}{m^2} \left| \nabla \tilde{\Psi} \right|^2 + \tilde{\alpha} |\tilde{\Psi}|^2 \sin^2 \tilde{\Phi} . \quad (5)$$

This equation is completely uncoupled from equation for \mathbf{v}_s . However, in the presence of half-quantum vortices there is a topological connection due to the Aharonov-Bohm effect: for the Bose condensate, the half-quantum vortex in the pair condensate is viewed as the Aharonov-Bohm tube with the half-quantum magnetic flux. Since ϕ has 2π winding around each half-quantum vortex, the phase $\tilde{\Phi}$ must have a π -jump across some line terminating on a half-quantum vortex. The structure of the whole system can be easily found in two extreme cases determined by the Josephson coupling $\tilde{\alpha}$.

If the Josephson coupling is big, the phase of the Bose-condensate $\tilde{\Phi}$ is strongly pinned by the phase ϕ of the pair condensate, and one has either $\tilde{\Phi} = 0$ or $\tilde{\Phi} = \pi$. In this case the π -jump is realized due to the π -soliton in Fig. 1(a). The Bose-condensate wave function can be represented as

$$\begin{aligned} \frac{\tilde{\Psi}(x, y)}{\sqrt{\rho_0}} = & (\Theta(-x - x_0) + \Theta(x - x_0)) \text{sign } y + \\ & + \Theta(x + x_0) \Theta(x_0 - x) \tanh \frac{y}{\xi} , \end{aligned} \quad (6)$$

where Θ is the step function; $(x, y) = (x_0, 0)$ and $(x, y) = (-x_0, 0)$ are positions of half-quantum vortices, which are topologically confined by the π -soliton; and ξ is the coherence length of the Bose-condensate. The phase $\tilde{\Phi}$ has π -jumps on the dashed lines terminating on half-quantum vortices in Fig. 1(a).

If the tension σ of the π -soliton is known, the distance $R = 2x_0$ between the Alice strings in such a non-axisymmetric vortex can be easily found from the consideration of the R -dependent part of the energy of the vortex per unit length:

$$U_{\text{vortex}}(R) = \sigma R - \frac{\pi}{2} \frac{\hbar^2}{m^2} \rho \ln R . \quad (7)$$

Here the first term is the energy of confinement of half-quantum vortices due to the tension of the soliton, and the second term is the hydrodynamic repulsion of half-quantum vortices. Minimization gives

the equilibrium distance:

$$R = \frac{\pi \hbar^2 \rho}{2\sigma m^2}. \quad (8)$$

For dilute Bose condensate, the coherence length of the Bose condensate is large compared with the coherence length of the pair condensate, $\xi \gg a$; the coherence length of the pair condensate a , which determines the core of the Alice string, is on the order of interatomic distance. Taking into account that the relative density of the Bose condensate $\rho_0/\rho \sim a^2/\xi^2$; the surface tension $\sigma \sim \hbar^2/(ma\xi^3)$; and $\rho \sim m/a^3$, one obtains $R/a \sim \xi^3/a^3 \sim (\rho/\rho_0)^{3/2} \gg 1$.

In the other extreme case, when the pinning of the Bose-condensate phase is weak, it is more advantageous to fix the order parameter magnitude $|\Psi| = \sqrt{\rho_0}$, and vary the phase $\tilde{\Phi}$. In this configuration instead of π -soliton, the Bose condensate contains the fat soliton with the $N = 1$ vortex in the center (Fig. 1(b)). Within the fat soliton the phase of the condensate changes from $\pm\pi/2$ to 0 in the region of thickness $\tilde{\xi} = \hbar/(m\tilde{\alpha}^{1/2})$. The tension of the fat soliton, which now enters the confinement term in Eq.(7), is $\sigma \sim \tilde{\xi}\tilde{\alpha}\rho_0 \sim \tilde{\alpha}^{1/2}\rho_0(\hbar/m) \sim \hbar^2/(ma\xi^2\tilde{\xi})$.

The weak-pinning regime occurs when $\tilde{\xi} \gg \xi$, while the strong-pinning regime occurs when $\tilde{\xi} \ll \xi$. In both regimes, and also in the intermediate regime when $\tilde{\xi} \sim \xi$, the structure of the vortex core is highly anisotropic, and the core size considerably exceeds the interatomic distance a .

In conclusion, if the idea that the dominating non-condensate particles form the pair-correlated state is valid for superfluid ^4He , vortices in superfluid ^4He must be highly anisotropic in the limit $\rho_0/\rho \rightarrow 0$.

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